Exercise 82

In Example 1.3.4 we arrived at a model for the length of daylight (in hours) in Philadelphia on the tth day of the year:

$$L(t) = 12 + 2.8\sin\left[\frac{2\pi}{365}(t-80)\right]$$

Use this model to compare how the number of hours of daylight is increasing in Philadelphia on March 21 and May 21.

Solution

The rate that the number of daylight hours is increasing is given by the derivative of $L(t)$.

$$\begin{aligned}
L'(t) &= \frac{dL}{dt} \\
&= \frac{d}{dt}\left\{12 + 2.8\sin\left[\frac{2\pi}{365}(t-80)\right]\right\} \\
&= \frac{d}{dt}(12) + \frac{d}{dt}\left\{2.8\sin\left[\frac{2\pi}{365}(t-80)\right]\right\} \\
&= (0) + 2.8\frac{d}{dt}\left\{\sin\left[\frac{2\pi}{365}(t-80)\right]\right\} \\
&= 2.8\left\{\cos\left[\frac{2\pi}{365}(t-80)\right]\cdot\frac{d}{dt}\left[\frac{2\pi}{365}(t-80)\right]\right\} \\
&= 2.8\left\{\cos\left[\frac{2\pi}{365}(t-80)\right]\cdot\left(\frac{2\pi}{365}\right)\right\} \\
&= \frac{28\pi}{1825}\cos\left[\frac{2\pi}{365}(t-80)\right]
\end{aligned}$$

March 21 is the 80th day of the year, and May 21 is the 141st day of the year. Evaluate $L'(t)$ at these values of t.

$$L'(80) = \frac{28\pi}{1825}\cos\left[\frac{2\pi}{365}(80-80)\right] \approx 0.0482\ \frac{\text{hours of daylight}}{\text{day}}$$

$$L'(141) = \frac{28\pi}{1825}\cos\left[\frac{2\pi}{365}(141-80)\right] \approx 0.0240\ \frac{\text{hours of daylight}}{\text{day}}$$

Based on this model, the number of daylight hours in Philadelphia is increasing twice as fast per day on March 21 as it is on May 21.